ENGG1500 Concept Review Session

The Engineering Peer Helper Program

March 3rd, 7-8pm

Week 1

Concepts:

- Matrices: linear combination of vectors in matrix form
    - Square Matrix
    - Zero Matrix: all zeros
    - Diagonal upper triangular:
    - Identity matrix
    - Square
- Vectors
    - Characteristics
    - Have both Magnitude and Direction
- Matrix Operations
    - Transpose-rows and columns swap |A|^T
    - Vector addition – add the elements in the vectors element wise {x}+{y} = {x1+y1, x2+y2}
        - Can only be applied to vectors that are in the same direction or have the same number of components
    - Scalar multiplication – multiply each element by the scalar

Important Theories or Formulas:

- Matrix form of linear equations → a is the coefficient matrix, x is the variable and b is the right-hand or constant vector
- Vector addition and multiplication

Tips:

- Practice problems
- Work to get quick at basic operations

Week 2

Concepts:

- REF – pivots in each row are to the left of the pivots in the row below (all zeros below), pivot is the first non-zero number in a row (ROW OPERATIONS!!!! WRITE THEM DOWN!!! KEEP TRACK!!!) – try to make zeros below the pivots
- RREF – Pivots are 1, unique
- When do we use REF vs RREF?
- RREF gives a unique value → a unique value is that there is only one solution (particular) when looking at the last row of the RREF matrix
- Matrix multiplication with a vector
  - the dimensions of the vector has to equal the number of columns in matrix

Important Theories or Formulas:

- REF and RREF row reduction steps
- Multiplying matrix with a vector

Tips:

- Keep practicing!!
- If you made an error identify where it was (may get marks for it)
- Order of row operations, make sure to be careful

Week 3

Concepts:

- Parametric From – When to make it: you have more variables than useful rows (Non zero rows in RREF) so it gives a family of solutions add (free varible)ER to it
- How to pick a free variable – in RREF (one column that doesn't reduce fully)
- Homogenous Form – when output vector b is zero Ax = 0
- Augmented Matrices – one matrix that contains A matrix and b vector (Ax = b), separated by a line
- Consistent- one or more solutions
- Inconsistent- do not have one solution (eg, one or more row looks like this 0 0 0 | 5, this would be inconsistent)

Important Theories or Formulas:

Free variables = unknowns - # equations

Tips:

- make sure to state all real elements when giving solution in parametric form

- Review class notes!!
- Study the application problems associated with the concepts
- Make sure to read the wording of questions carefully
- Cover applications
- Questions can be changed and become more complex

Week 4

Concepts:

- Span and spanning set

2. Vector spanning (p. 18-19)

2-1 Spanning set and Span

Subspaces formed by the linear combination of vector sets

Theorem

If $\{\vec{v}_1, \dots, \vec{v}_k\}$ is a set of vectors in $\mathbb{R}^n$, then

$$S = \{t_1\vec{v}_1 + \cdots + t_k\vec{v}_k \mid t_1, \dots, t_k \in \mathbb{R}\}.$$

is a subspace of $\mathbb{R}^n$

spanning set $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \end{bmatrix} \right\}$

$S = \left\{ t_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + t_2 \begin{bmatrix} 3 \\ 5 \end{bmatrix} + t_3 \begin{bmatrix} -2 \\ 0 \end{bmatrix} \right\}$

B span S

↳ "is creating/is the base"

Definition of Span

Definition (Span)

$$S = \{t_1\vec{v}_1 + \cdots + t_k\vec{v}_k \mid t_1, \dots, t_k \in \mathbb{R}\},$$

is called the subspace spanned by $B = \{\vec{v}_1, \dots, \vec{v}_k\}$, and we say that B spans S.

The set B is called a spanning set for S. We denote S by

$$S = \text{Span}\{\vec{v}_1, \dots, \vec{v}_k\} = \text{Span}\, B.$$

$S = \text{span}\underbrace{\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots \vec{v}_n\}}_{B}$

↳ linear combination of those vectors

$S = \text{span}\, B$

$\vec{x} = t_1\vec{v}_1 + t_2\vec{v}_2 + t_3\vec{v}_3 + \cdots + t_n\vec{v}_n$

- Subspaces
    - A non-empty subset S of R^n is called a subspace of R^n if for all vectors x, y E S and t E R
    - Under linear combinations
- Column space

3.1 Column spaces

Definition (Columnspace)

The columnspace of an $m \times n$ matrix A is the set

subspace

$$\text{Col}(A) = \{A\vec{x} \in \mathbb{R}^m \mid x \in \mathbb{R}^n\}.$$

Alternative expression of the column space of a matrix A:

$$For\ A = [\vec{a}_1 \cdots \vec{a}_n], A \in R^{m \times n}, \vec{a}_1 \cdots \vec{a}_n \in R^m,$$

$$Col(A) = span\{\vec{a}_1 \cdots \vec{a}_n\} = \{Ax \ \in R^m\}$$

- Null space

Theorem/Definition

Let A be an $m \times n$ matrix. The set

$$S = \{\vec{x} \in \mathbb{R}^n \mid A\vec{x} = \vec{0}\}$$

of all solutions to a homogeneous system $A\vec{x} = \vec{0}$ is a subspace of $\mathbb{R}^n$. It is called the solution space of the system.

The nullspace of an $m \times n$ matrix A is

$$\text{Null}(A) = \{\vec{x} \in \mathbb{R}^n \mid A\vec{x} = \vec{0}\}.$$

- Basis
- Rank
    - Number of linearly independent rows in a matrix
- Linearly independent- none of the vectors are linear combinations of other vectors
- Linearly dependent- any vector in a set is a linear combination of any other vectors
- Dimension

Important Theories or Formulas:

- For rank → # of pivots when in RREF
    - Rank(A) = n or Ax=0 has only the trivial solution

Tips:

- YouTube resources for visualizing (3blue1brown)
- These concepts are good proof types of problems → making a mind map can be helpful
- Use your textbook for proofs

Week "5"

Concepts:

MIDTERM WEEK!!!

- Col(A)
- Row(A)
- Null(A)

3.2 Determination of bases for row spaces, column spaces, and null spaces (p. 157)

Basis of row space RREF

Theorem

Let B be the reduced row echelon form of a matrix A. Then the non-zero rows of B form a basis for Row(A), and hence the dimension of Row(A) equals the rank of A.

Basis of column space REF

Theorem

Suppose that B is the reduced echelon form of A. Then the columns of A that correspond to the columns of B with leading 1s form a basis of the columnspace of A. Hence, the dimension of the columnspace equals the rank of A. go back & get vectors from original matrix

Bases of Null Spaces

The Spanning set of the general solution of $A\vec{x} = \vec{0}$ *, is a basis of* $Null(A)$ (is a subspace)

Dimensions of subspaces count the # of bases

Definition (Dimension)

If a vector space $\mathbb{V}$ has a basis with n vectors, then we say that the dimension of $\mathbb{V}$ is n and write $\dim \mathbb{V} = n$.

The dimension of the trivial vector space $\{\mathbf{0}\}$ is defined to be 0.

Definition (Nullity)

The dimension of the nullspace of a matrix A is the nullity of A and is denoted by nullity(A).

Important Theories or Formulas:

- Refer to page 157 in your textbook (may vary with edition number)
- REF and RREF

Tips:

- Go over theories in the textbook to help understanding concepts that could be asked as a proof question
- Try to make a mind map or concept review → relating concepts together to try and understand what they mean and how they relate to one another

Problem Solving Strategies

A^{3x3} = calibration matrix

$$\begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 3 \\ 10 \end{bmatrix}$$

Therefore,

$$A = \begin{bmatrix} 7 & 14 & 15 \\ 2 & 4 & 3 \\ 3 & 6 & 10 \end{bmatrix}$$

Question is asking us to determine if there is only one way to combine the three column vectors of A into a relation of linear dependence. To do this, can check if the homogenous problem has only a trivial solution.

$x_1 = -2 x_2$
$x_3 = 0$
x_2 is a free variable

Therefore, there an infinite number of solutions to the homogenous problem, so the sensor's requirement is not met.

Questions and Contact

a) This will be posted on The Engineering Peer Helpers (EPH) Website.

   i) https://www.uoguelph.ca/engineering/content/current/peer-helper

b) There will not be a filled in version posted. Please write notes during the session.

c) Stay tuned for more ENGG*1500 workshops/sessions before the final exam.

d) Email for a small-group consultation. It’s great to think of your questions and send them beforehand!

e) Book a one-on-one consultation for ENGG*1500!